Resonances in finite-resource interconversion

Christopher Chubb
EQus, University of Sydney

Kamil Korzekwa (USyd→UGdańsk)
Marco Tomamichel (UTS)

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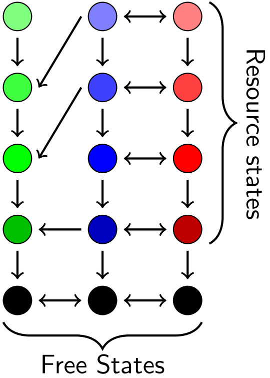
Resource Theories

- Resource theories describe the physics of constrained systems
- Defined by sets of free states and free operations
- Any non-free state is a *resource state*
- How do we quantify the resourcefulness of a state?



Resource theories

Resource theory	Physical constraint	Free operations	Free states
Entanglement	Physical separation	Local operations and classical communication	Separable states
Thermodynamics	Laws of Thermodynamics	Thermal operations	Thermal state
Coherence	Inability to create coherent superpositions in a specific basis	Incoherent operations	Incoherent states
Purity	Inability to purify states	Unital operations	Maximally mixed state

Entanglement (bipartite)

Separable states:

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

Local ops. and classical communication:

$$\mathcal{E}(\rho) = \sum_a (\mathcal{I} \otimes \mathcal{E}_a)\Big((M_a \otimes I)\rho(M_a^\dagger \otimes I)\Big)$$

Thermodynamics

Thermal state:

$$\gamma = e^{-\beta H} / \operatorname{Tr} e^{-\beta H}$$

Thermal operations:

$$\mathcal{E}(\rho) = \operatorname{Tr}_B U(\rho \otimes \gamma^k)U^\dagger \qquad [U, H] = 0$$

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Resource interconversion problem

One way of assessing the resourcefulness of a state is to ask which resource interconversions are possible.

Single-shot

For which states ρ and σ do there exist free operations $\mathcal{E}$ such that

$$\mathcal{E}(\rho) = \sigma \quad \text{or} \quad \mathcal{E}(\rho) \approx_\epsilon \sigma.$$

- Conditions may be difficult to compute
- Doesn't tell us about large numbers of states

Asymptotic

What is the maximum rate $R(\rho, \sigma)$ such that

$$\mathcal{E}\left(\rho^{\otimes n}\right) \approx \sigma^{\otimes Rn}$$

as $n \to \infty$.

- $R(\rho, \sigma)$ often easy to compute
- Obscures finite-size or finite-error effects

Intermediate

We will consider the intermediate regime: rates $R(n, \epsilon)$, with $n < \infty$ and $\epsilon > 0$, such that

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Majorisation-based resource theories

Entanglement

Restriction: bipartite pure-state

$$\rho = |\psi\rangle\langle\psi| \qquad \sigma = |\phi\rangle\langle\phi|$$

States represented by Schmidt coefficients:

$$|\psi\rangle = \sum_i \sqrt{p_i}\,|l_i\rangle \otimes |r_i\rangle$$
$$|\phi\rangle = \sum_i \sqrt{q_i}\,|l_i'\rangle \otimes |r_i'\rangle$$

Majorisation:

$$\boldsymbol{p} \succ \boldsymbol{q} \quad \Leftrightarrow \quad \sum_{i=1}^{k} p_i^{\downarrow} \geq \sum_{i=1}^{k} q_i^{\downarrow} \ \forall k$$

Thermodynamics

Restriction: energy-incoherent states

$$[\rho, H] = [\sigma, H] = 0$$

States represented by spectra:

$$\rho = \sum_i p_i\,|E_i\rangle\langle E_i|$$
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Notation

Entanglement

In the entanglement case our results rely on the entropy and entropy variance:

$$H(\boldsymbol{p}) := -\sum_i p_i \log p_i = S(\mathrm{Tr}_2 \, |\psi\rangle\langle\psi|)$$

$$H(\boldsymbol{q}) := -\sum_i q_i \log q_i = S(\mathrm{Tr}_2 \, |\phi\rangle\langle\phi|)$$

$$V(\boldsymbol{p}) := \sum_i p_i \left(\log p_i + H(\boldsymbol{p})\right)^2 = V(\mathrm{Tr}_2 \, |\psi\rangle\langle\psi|)$$

$$V(\boldsymbol{q}) := \sum_i p_i \left(\log q_i + H(\boldsymbol{q})\right)^2 = V(\mathrm{Tr}_2 \, |\phi\rangle\langle\phi|)$$

Thermodynamics

In the thermo case our results rely on the relative entropy and relative entropy variance:

$$D(\boldsymbol{p}\|\boldsymbol{\gamma}) := \sum_i p_i \log \frac{p_i}{\gamma_i} = D(\rho\|\gamma)$$

$$D(\boldsymbol{q}\|\boldsymbol{\gamma}) := \sum_i q_i \log \frac{q_i}{\gamma_i} = D(\sigma\|\gamma)$$

$$V(\boldsymbol{p}\|\boldsymbol{\gamma}) := \sum_i p_i \left(\log p_i + H(\boldsymbol{p})\right)^2 = V(\rho\|\gamma)$$

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Resource interconversion: Single-shot

Nielsen's Theorem

There exists an LOCC transformation $|\psi\rangle \to |\phi\rangle$ iff $\boldsymbol{p} \prec \boldsymbol{q}$, where

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There exists an TO transformation $\rho \to \sigma$ iff $\boldsymbol{p} \succ^{\beta} \boldsymbol{q}$, where

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Resource interconversion: Asymptotic

Entanglement

The asymptotic rate of LOCC transformation is

$$R_\infty^{(\mathrm{ent})}\left(|\psi\rangle \to |\phi\rangle\right) = \frac{H(\boldsymbol{p})}{H(\boldsymbol{q})}$$

Thermodynamics

The asymptotic rate of TO transformation is

$$R_\infty^{(\mathrm{th})}\left(\rho \to \sigma\right) = \frac{D(\boldsymbol{p}\|\gamma)}{D(\boldsymbol{q}\|\gamma)}$$

Both theories are asymptotically reversible

$$R_\infty(\rho \to \sigma) \cdot R_\infty(\sigma \to \rho) = 1.$$

Resourcefulness entirely captured by the entanglement entropy/free energy

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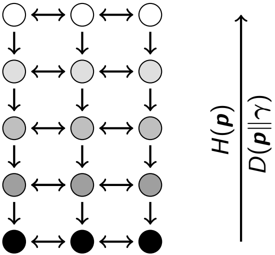

Resource interconversion: Intermediate

Small-deviation[1,2]

For $\epsilon \in (0,1)$

$$R_n^{(\mathrm{ent})} = \frac{H(\boldsymbol{p}) + \sqrt{\frac{V(\boldsymbol{p})}{n}} \cdot f_\nu(\epsilon)}{H(\boldsymbol{q})} + o(1/\sqrt{n}) \qquad \text{where } \nu := \frac{V(\boldsymbol{p})/H(\boldsymbol{p})}{V(\boldsymbol{q})/H(\boldsymbol{q})}$$

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Moderate-deviation[3]

For ϵ_n shrinking non-exponentially, $\epsilon_n = e^{-n^\alpha}$ with $\alpha \in (0,1)$,

$$R_n^{(\mathrm{ent})} = \frac{H(\boldsymbol{p}) - \sqrt{2V(\boldsymbol{p})}\left|1 - 1/\sqrt{\nu}\right| \cdot n^{-(1-\alpha)/2}}{H(\boldsymbol{q})} + o\left(n^{-(1-\alpha)/2}\right)$$

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Resource resonance

- All of these expressions exhibit irreversibility for low error

$$R_n(\rho \to \sigma) \cdot R_n(\sigma \to \rho) \leq 1$$

- The quantity ν quantifies the magnitude of the finite-size effect (up to second order),

$$\nu = 1 \quad \Longleftrightarrow \quad R_n(\rho \to \sigma) \cdot R_n(\rho \to \sigma) = 1$$

- By tuning out states such that $\nu = 1$, we can mitigate finite-size effects

Resonance example 1: Entanglement

We have access to copies of $|\psi_1\rangle$ or $|\psi_2\rangle$, and want to create copies of $|\phi\rangle$, where

$$H(\boldsymbol{p}_1) = H(\boldsymbol{q}) = H(\boldsymbol{p}_2)$$
$$V(\boldsymbol{p}_1) < V(\boldsymbol{q}) < V(\boldsymbol{p}_2).$$

Asymptotically we expect $R_\infty = 1$ for either $|\psi_1\rangle$ or $|\psi_2\rangle$.

To lower the error for a fixed n, we should pick a resonant input state

$$|\psi_1\rangle^{\otimes \lambda n} \otimes |\psi_2\rangle^{\otimes (1-\lambda) n}$$

$$\lambda \approx \frac{V(\boldsymbol{q}) - V(\boldsymbol{p}_2)}{V(\boldsymbol{p}_1) - V(\boldsymbol{p}_2)}$$

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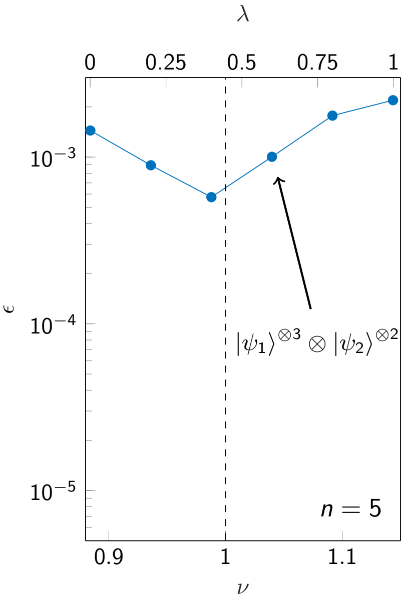

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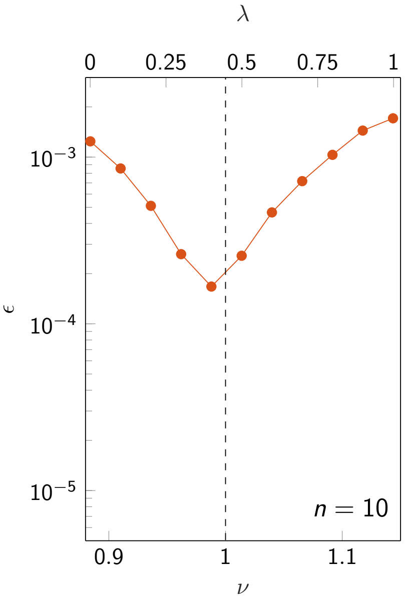

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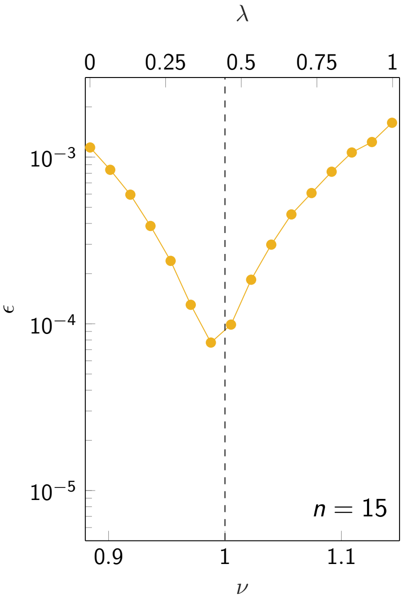

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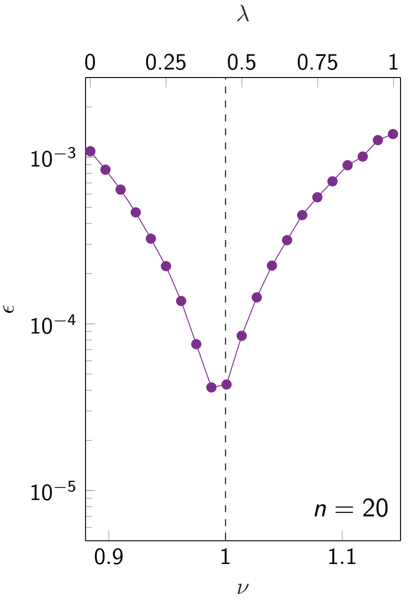

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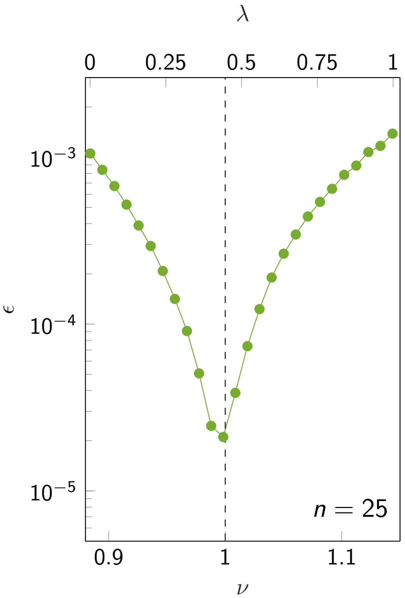

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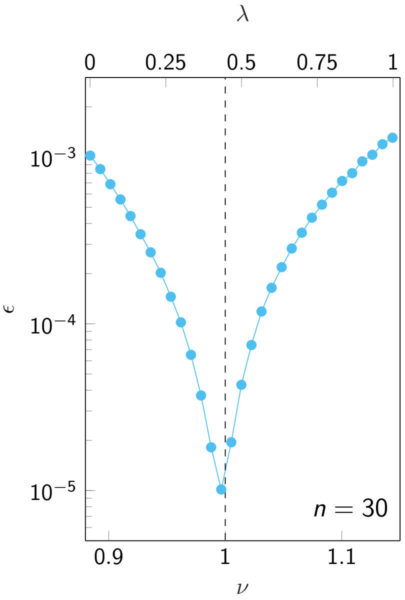

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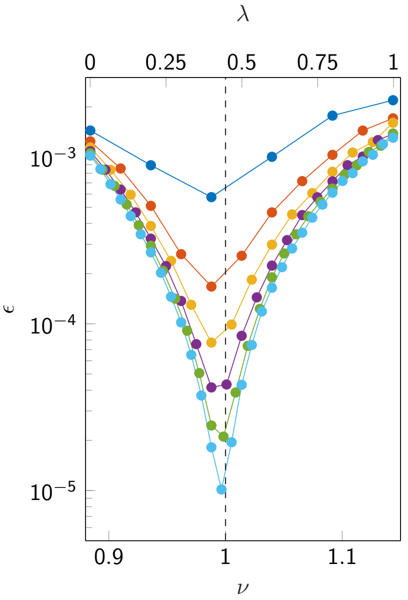
To lower the error for a fixed n, we should pick a resonant input state

$$|\psi_1\rangle^{\otimes \lambda n} \otimes |\psi_2\rangle^{\otimes (1-\lambda) n}$$

$$\lambda \approx \frac{V(\boldsymbol{q}) - V(\boldsymbol{p}_2)}{V(\boldsymbol{p}_1) - V(\boldsymbol{p}_2)}$$

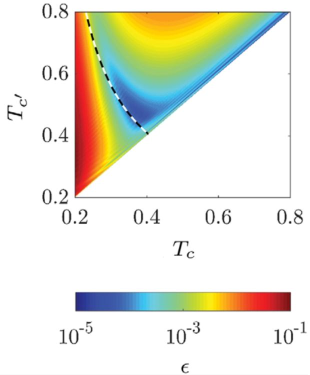


Resonance example 2: Thermodynamics

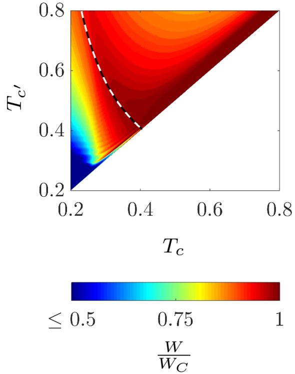
Consider a heat engine, with a working body of $n = 200$ qubits.

There is a cold bath at temperature T_c and hot bath at temperature $T_h = 1$, and the engine is operated between $T_c \leftrightarrow T_{c'}$.

$W = .95 \cdot W_C$



$\epsilon = 10^{-3}$



Conclusions and further work

- We have investigated a resonance phenomena in majorisation-based resonance theorems
- Can this be used in near-term/NISQ experiments?
- Can we can drop the restrictions (mixed-state entanglement, energy-coherent thermo)?
- Does an analogous phenomenon occur for non-majorisation-based resource theories (e.g. magic)?

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Thank you!

me@christopherchubb.com
christopherchubb.com
@QuantumChubb