Resonances in finite-resource interconversion

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Small: 1711.01193 10/c7tt (Quantum)

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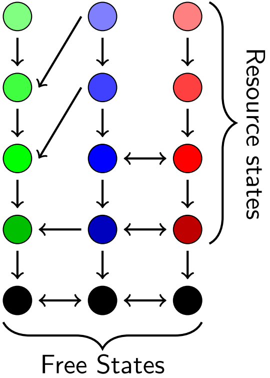
Resource Theories

- Resource theories describe the physics of constrained systems
- Defined by sets of free states and free operations
- Any non-free state is a *resource state*
- How do we quantify the resourcefulness of a state?



Resource Theories

Resource theory	Physical constraint	Free operations	Free states
Entanglement	Physical separation	Local operations and classical communication	Separable states
Thermodynamics	Laws of Thermodynamics	Thermal operations	Thermal state
Coherence	Inability to create coherent superpositions in a specific basis	Incoherent operations	Incoherent states
Purity	Inability to purify states	Unital operations	Maximally mixed state

Entanglement (bipartite)

Separable states:

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

Local ops. and classical communication:

$$\mathcal{E}(\rho) = \sum_a (\mathcal{I} \otimes \mathcal{E}_a)\Big((M_a \otimes I)\rho(M_a^\dagger \otimes I)\Big)$$

Thermodynamics

Thermal state:

$$\gamma = e^{-\beta H} / \operatorname{Tr} e^{-\beta H}$$

Thermal operations:

$$\mathcal{E}(\rho) = \operatorname{Tr}_B U(\rho \otimes \gamma^k)U^\dagger \quad [U, H] = 0$$

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Resource interconversion problem

One way of assessing the resourcefulness of a state is to ask which resource interconversions are possible.

Single-shot

For which states ρ and σ do there exist free operations $\mathcal{E}$ such that

$$\mathcal{E}(\rho) = \sigma \quad \text{or} \quad \mathcal{E}(\rho) \approx_\epsilon \sigma.$$

- Conditions may be difficult to compute
- Doesn't tell us about large numbers of states

Asymptotic

What is the maximum rate R such that

$$\mathcal{E}\left(\rho^{\otimes n}\right) \approx \sigma^{\otimes Rn}$$

as $n \to \infty$.

- R often easier to compute
- Obscures finite-size or finite-error effects

Intermediate

We will consider the intermediate regime: rates $R(n, \epsilon)$, with $n < \infty$ and $\epsilon > 0$, such that

$$\mathcal{E}\left(\rho^{\otimes n}\right) \approx_\epsilon \sigma^{\otimes Rn}.$$

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Majorisation-based resource theories

Entanglement

Restriction: bipartite pure-state

$$\rho = |\psi\rangle\langle\psi| \qquad \sigma = |\phi\rangle\langle\phi|$$

States represented by Schmidt coefficients:

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |l_i\rangle \otimes |r_i\rangle$$
$$|\phi\rangle = \sum_i \sqrt{q_i}\, |l_i'\rangle \otimes |r_i'\rangle$$

Majorisation:

$$\boldsymbol{p} \succ \boldsymbol{q} \iff \sum_{i=1}^{k} p_i^{\downarrow} \geq \sum_{i=1}^{k} q_i^{\downarrow} \; \forall k$$

Thermodynamics

Restriction: energy-incoherent states

$$[\rho, H] = [\sigma, H] = 0$$

States represented by spectra:

$$\rho = \sum_i p_i\, |E_i\rangle\langle E_i|$$
$$\sigma = \sum_i q_i\, |E_i'\rangle\langle E_i'|$$
$$\gamma = \sum_i \gamma_i\, |E_i''\rangle\langle E_i''|$$

Thermo-majorisation:

$$\boldsymbol{p} \succ^{\beta} \boldsymbol{q} \iff \sum_{i=1}^{k} p_i^{\downarrow} e^{-\beta E_i} \geq \sum_{i=1}^{k} q_i^{\downarrow} e^{-\beta E_i} \; \forall k$$
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Resource interconversion: Single-shot

Nielsen's theorem

There exists an LOCC transformation $|\psi\rangle \to |\phi\rangle$ iff $\boldsymbol{p} \prec \boldsymbol{q}$, where

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Thermo-Nielsen's theorem

There exists an TO transformation $\rho \to \sigma$ iff $\boldsymbol{p} \succ^{\beta} \boldsymbol{q}$, where

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Approximate majorisation

If we only require approximate interconversion we need to relax the notion of majorisation as well:

- A distribution $\boldsymbol{p}$ ϵ-pre-majorises a distribution $\boldsymbol{q}$, which we denote $\boldsymbol{p}\,{}_{\epsilon}\!\succ \boldsymbol{q}$, if there exists a $\tilde{\boldsymbol{p}}$ such that

$$\tilde{\boldsymbol{p}} \succ \boldsymbol{q} \qquad \text{and} \qquad \delta(\boldsymbol{p}, \tilde{\boldsymbol{p}}) \leq \epsilon.$$

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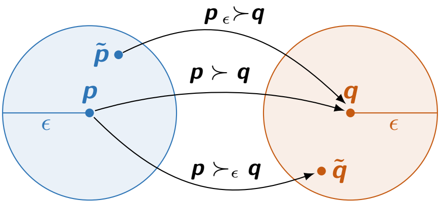

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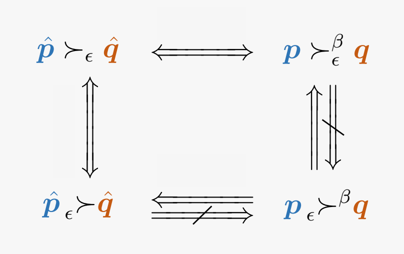

Approximate Nielsen's theorem

There exists an LOCC transformation $|\psi\rangle \to |\tilde{\phi}\rangle$ with $|\langle\phi|\tilde{\phi}\rangle|^2 \geq 1 - \epsilon$ iff $\boldsymbol{p} \prec_{\epsilon} \boldsymbol{q}$.

And similar for thermal majorisation...

Asymptotic notation

Entanglement

In the entanglement case our results rely on the entropy and varentropy:

$$\begin{aligned} H(\boldsymbol{p}) &:= -\sum_i p_i \log p_i \\ &= S(\mathrm{Tr}_2 |\psi\rangle\langle\psi|) \\ V(\boldsymbol{p}) &:= \sum_i p_i \left(\log p_i + H(\boldsymbol{p})\right)^2 \\ &= V(\mathrm{Tr}_2 |\psi\rangle\langle\psi|) \end{aligned}$$

Thermodynamics

In the thermo case our results rely on the relative entropy and relative varentropy:

$$\begin{aligned} D(\boldsymbol{p}\|\gamma) &:= \sum_i p_i \log \frac{p_i}{\gamma_i} \\ &= D(\rho\|\gamma) \\ V(\boldsymbol{p}\|\gamma) &:= \sum_i p_i \left(\log p_i + H(\boldsymbol{p})\right)^2 \\ &= V(\rho\|\gamma) \end{aligned}$$

Resource interconversion: Asymptotic

Entanglement

The asymptotic rate of LOCC transformation is

$$R_\infty^{(\mathrm{ent})}\left(|\psi\rangle \to |\phi\rangle\right) = \frac{H(\boldsymbol{p})}{H(\boldsymbol{q})}$$

Thermodynamics

The asymptotic rate of TO transformation is

$$R_\infty^{(\mathrm{th})}\left(\rho \to \sigma\right) = \frac{D(\boldsymbol{p}\|\gamma)}{D(\boldsymbol{q}\|\gamma)}$$

Both theories are asymptotically reversible

$$R_\infty(\rho \to \sigma) \cdot R_\infty(\sigma \to \rho) = 1.$$

Resourcefulness entirely captured by the entanglement entropy/free energy

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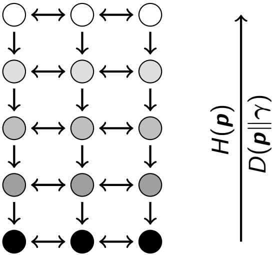
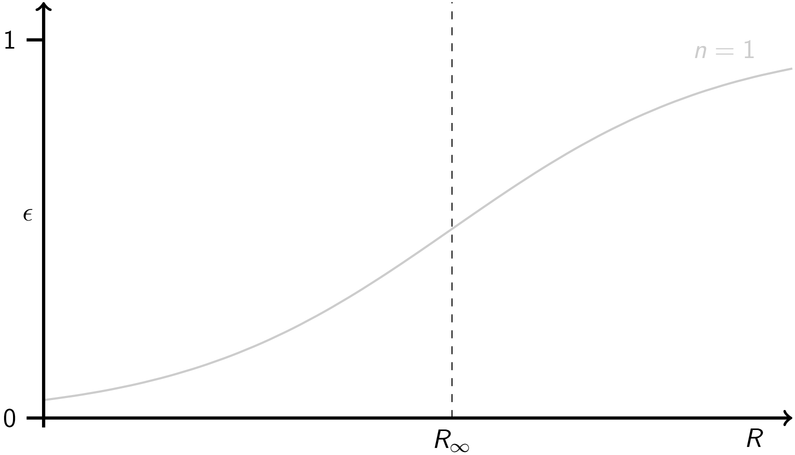


Deviation analyses

We know what R_∞ is, but how does $R_n \to R_\infty$?



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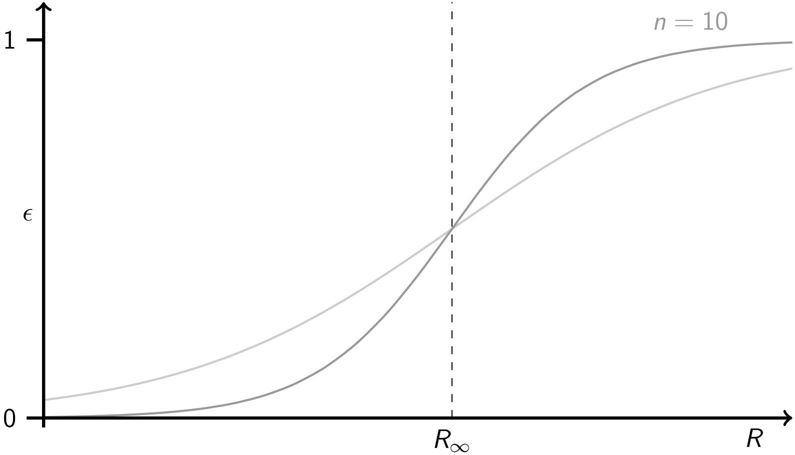

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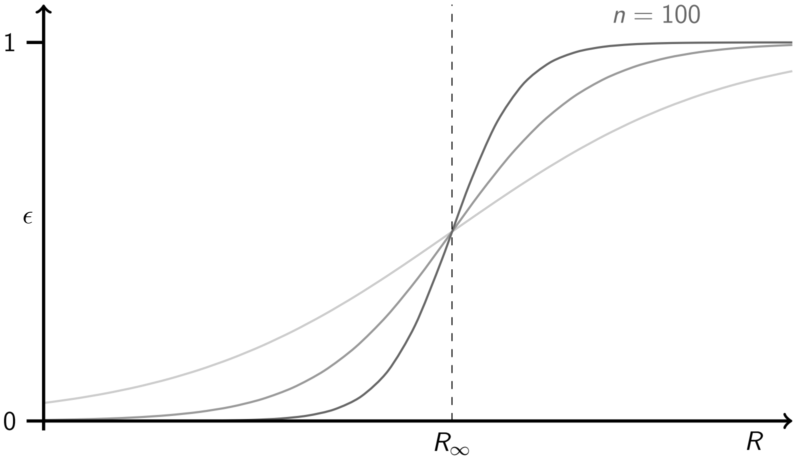

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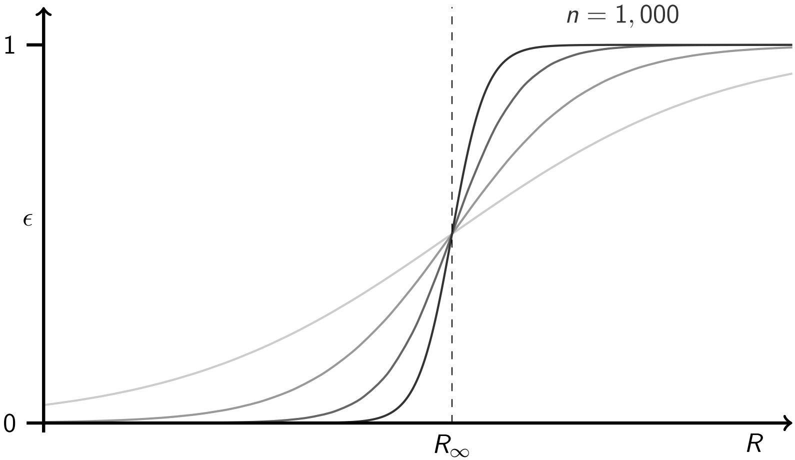

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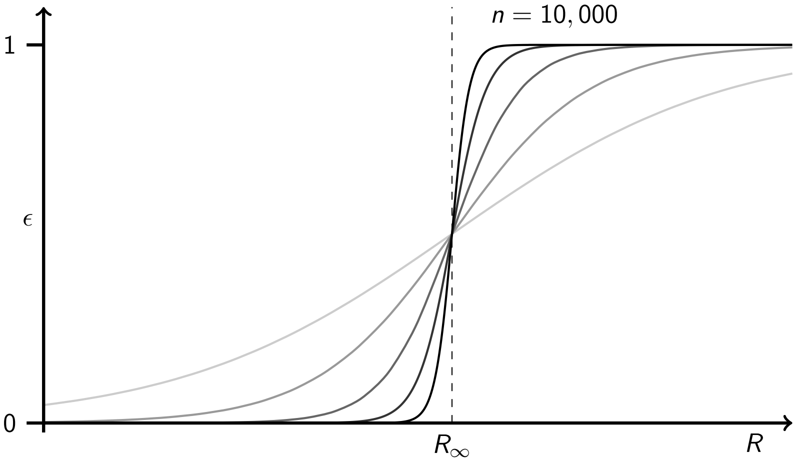

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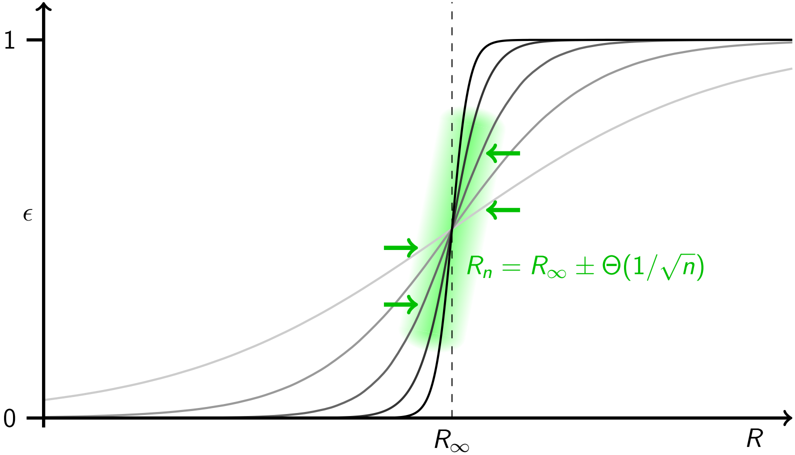

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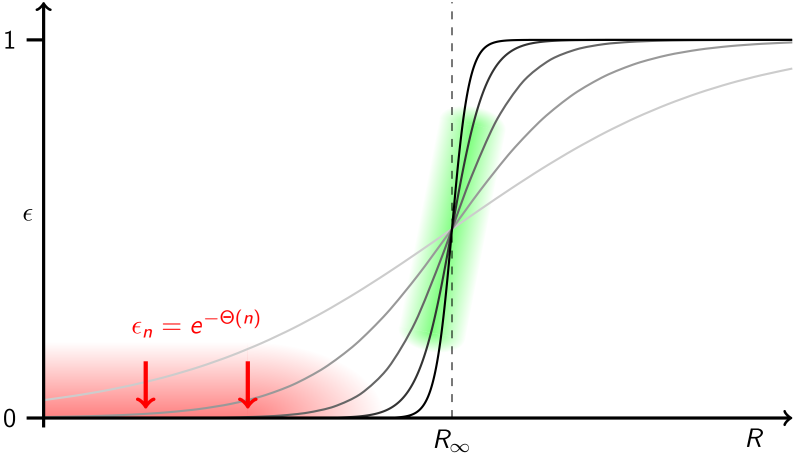

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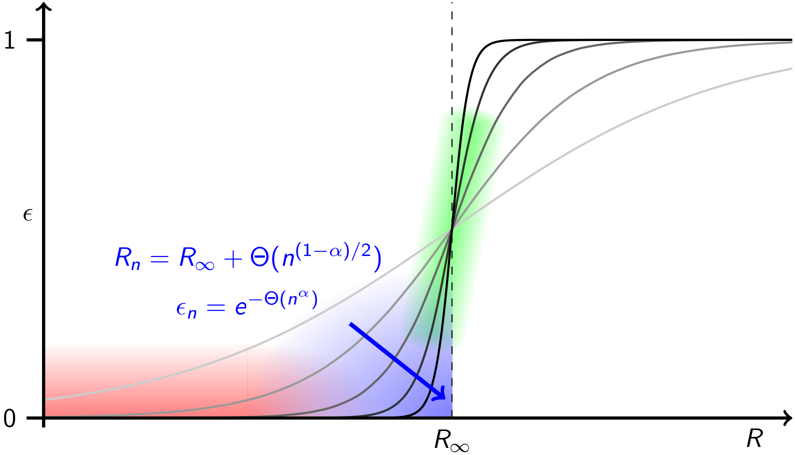


Resource interconversion: Small deviations

Small-deviation[1,2]

For $\epsilon \in (0,1)$

$$R_{n,\epsilon}^{(\mathrm{ent})} = \frac{H(\boldsymbol{p}) + \sqrt{\frac{V(\boldsymbol{p})}{n}} \cdot Z_\nu^{-1}(\epsilon)}{H(\boldsymbol{q})} + +o\left(\frac{1}{\sqrt{n}}\right), \qquad \nu := \frac{V(\boldsymbol{p})/H(\boldsymbol{p})}{V(\boldsymbol{q})/H(\boldsymbol{q})},$$

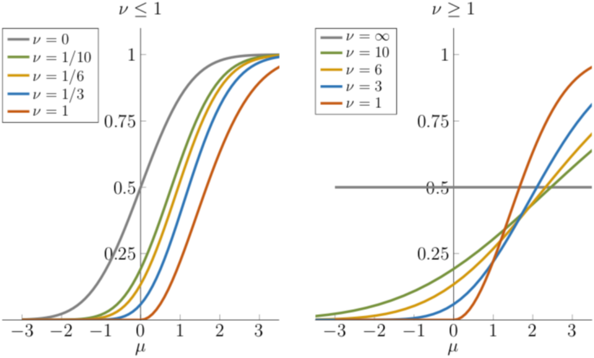
$$R_{n,\epsilon}^{(\mathrm{th})} = \frac{D(\boldsymbol{p}\|\gamma) + \sqrt{\frac{V(\boldsymbol{p}\|\gamma)}{n}} \cdot Z_\nu^{-1}(\epsilon)}{D(\boldsymbol{q}\|\gamma)} + o\left(\frac{1}{\sqrt{n}}\right), \quad \nu := \frac{V(\boldsymbol{p}\|\gamma)/D(\boldsymbol{p}\|\gamma)}{V(\boldsymbol{q}\|\gamma)/D(\boldsymbol{q}\|\gamma)}.$$

Properties of the Rayleigh-normal distribution:

- We have $Z_\nu^{-1}(\epsilon) > 0$ for all $\epsilon > 0$ if $\nu = 1$.
- We have $Z_\nu^{-1}(\epsilon) = \Phi^{-1}(\epsilon)$ if $\nu = 0$, i.e. if $V(p) = 0$. This happens for example when we convert from a maximally entangled state.

[1] Kumagai, Hayashi, arXiv:1306.4166, doi:10/f9tvhb (IEEE TIT)
[2] Chubb, Tomamichel, Korzekwa, arXiv:1711.01193, doi:10/c7tt (Quantum)

Rayleigh-normal distribution: $Z_\nu(\mu)$



The formal defintion:[1]

$$Z_\nu(\mu) := 1 - \sup_{A \geq \Phi} \mathcal{F}\left(A', \Phi'_{\mu,\nu}\right),$$

where the supremum is taken over all monotone increasing and continuously differentiable $A : \mathbb{R} \to [0,1]$ such that $A \geq \Phi$ pointwise.

Resource interconversion: Moderate deviations

- Often we care more about the case of asymptotically vanishing error.
- Good news: we do not need to deal with Rayleigh-normal distributions!

Moderate-deviation[3]

For ϵ_n shrinking non-exponentially, $\epsilon_n = e^{-n^\alpha}$ with $\alpha \in (0,1)$,

$$R^{(\mathrm{ent})}_{n,\epsilon_n} = \frac{H(\boldsymbol{p}) - \sqrt{2V(\boldsymbol{p})}\,|1-1/\sqrt{\nu}| \cdot n^{-(1-\alpha)/2}}{H(\boldsymbol{q})} + o\left(n^{-(1-\alpha)/2}\right)$$

$$R^{(\mathrm{th})}_{n,\epsilon_n} = \frac{D(\boldsymbol{p}\|\gamma) - \sqrt{2V(\boldsymbol{p}\|\gamma)}\,|1-1/\sqrt{\nu}| \cdot n^{-(1-\alpha)/2}}{D(\boldsymbol{q}\|\gamma)} + o\left(n^{-(1-\alpha)/2}\right)$$

[3] Chubb, Tomamichel, Korzekwa, arXiv:1809.07778, doi:10/gfxbhd (PRA)

Resource resonance

- All of these expressions exhibit irreversibility for low error

$$R_{n,\epsilon}(\rho \to \sigma) \cdot R_{n,\epsilon}(\sigma \to \rho) < 1$$

- The quantity ν quantifies the magnitude of the finite-size effect (up to second order),

$$\nu = 1 \quad \Longleftrightarrow \quad R_{n,\epsilon}(\rho \to \sigma) \cdot R_{n,\epsilon}(\rho \to \sigma) = 1$$

- By tuning out states such that $\nu = 1$, we can mitigate finite-size effects

Resonance example 1: Entanglement

We have access to copies of $|\psi_1\rangle$ or $|\psi_2\rangle$, and want to create copies of $|\phi\rangle$, where

$$H(\boldsymbol{p}_1) = H(\boldsymbol{q}) = H(\boldsymbol{p}_2)$$
$$V(\boldsymbol{p}_1) < V(\boldsymbol{q}) < V(\boldsymbol{p}_2).$$

Asymptotically we expect $R_\infty = 1$ for either $|\psi_1\rangle$ or $|\psi_2\rangle$.

To lower the error for a fixed n, we should pick a resonant input state

$$|\psi_1\rangle^{\otimes \lambda n} \otimes |\psi_2\rangle^{\otimes (1-\lambda) n}$$

$$\lambda \approx \frac{V(\boldsymbol{q}) - V(\boldsymbol{p}_2)}{V(\boldsymbol{p}_1) - V(\boldsymbol{p}_2)}$$

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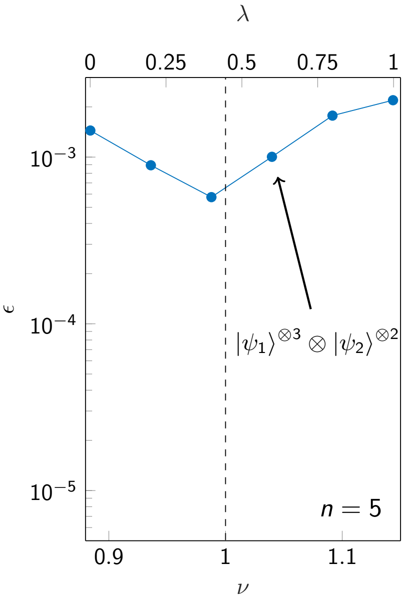

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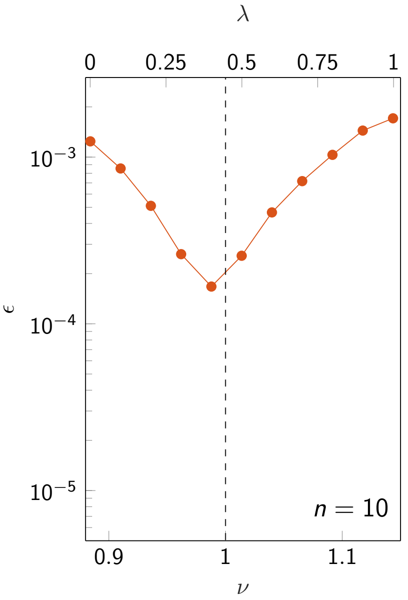

Resonance example 1: Entanglement

We have access to copies of $|\psi_1\rangle$ or $|\psi_2\rangle$, and want to create copies of $|\phi\rangle$, where

$$H(\boldsymbol{p}_1) = H(\boldsymbol{q}) = H(\boldsymbol{p}_2)$$
$$V(\boldsymbol{p}_1) < V(\boldsymbol{q}) < V(\boldsymbol{p}_2).$$

Asymptotically we expect $R_\infty = 1$ for either $|\psi_1\rangle$ or $|\psi_2\rangle$.

To lower the error for a fixed n, we should pick a resonant input state

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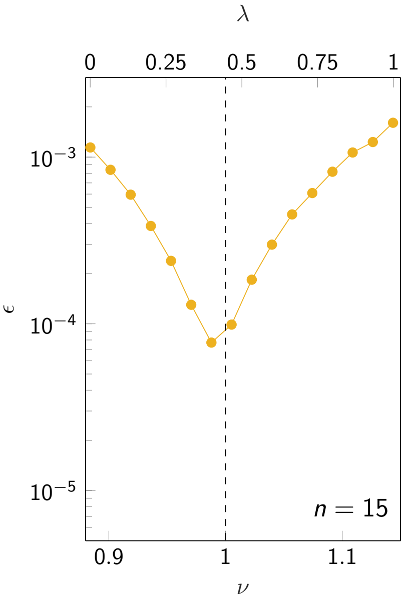

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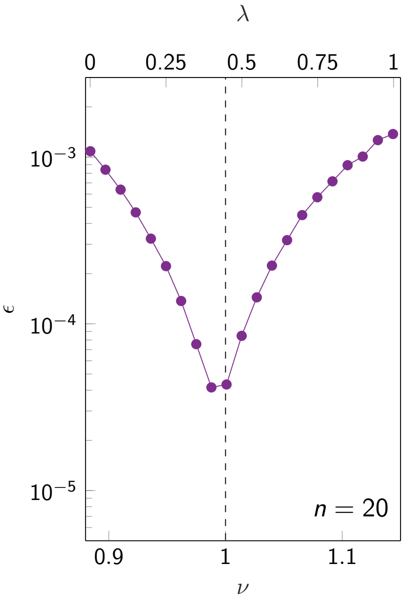

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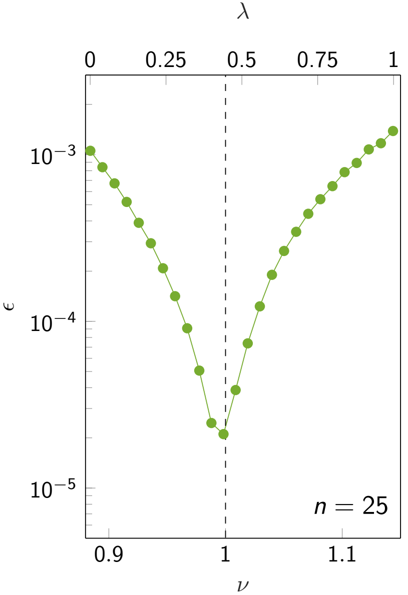

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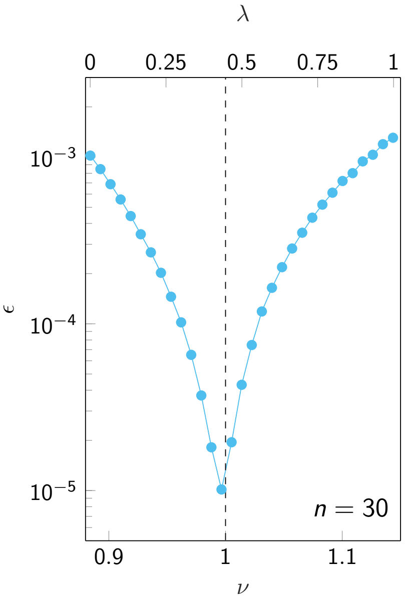

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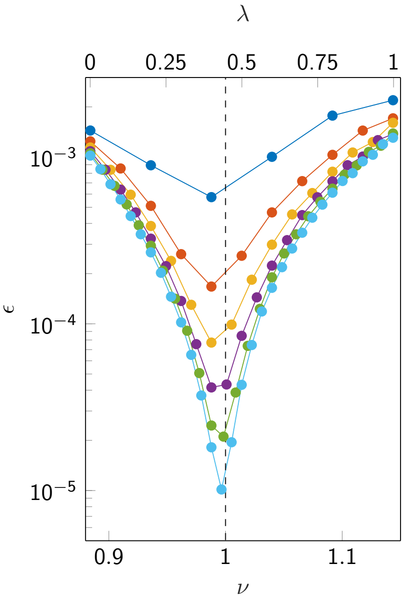
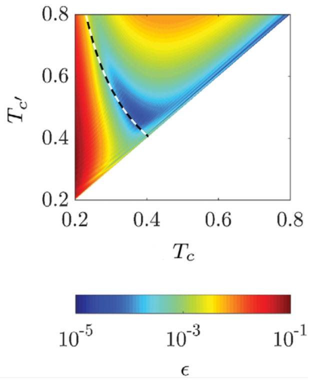


Resonance example 2: Thermodynamics

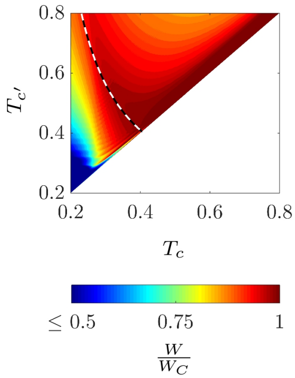
Consider a heat engine, with a working body of $n = 200$ qubits.

There is a cold bath at temperature T_c and hot bath at temperature $T_h = 1$, and the engine is operated between $T_c \leftrightarrow T_{c'}$.

$W = .95 \cdot W_C$

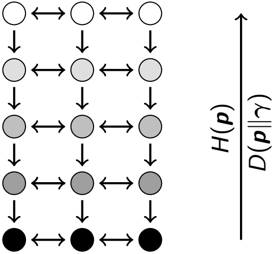


$\epsilon = 10^{-3}$

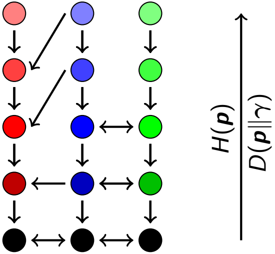


Structure of asymptotically reversible resource theories

If we only look at first order asymptotics:



If we look at higher order asymptotics (second-order or moderate deviation):



Conclusions and further work

- We have investigated a resonance phenomena in majorisation-based resource theories.
- Can this be used in near-term/NISQ experiments?
- Can we can drop some of the restrictions (e.g. energy-coherent thermo, mixed-state entanglement)?
- Does an analogous phenomenon occur for non-majorisation-based resource theories (e.g. magic)?

Resonance: 1810.02366 10/gfxb5z (PRL)
Moderate: 1809.07778 10/gfxbhd (PRA)
Small: 1711.01193 10/c7tt (Quantum)

Thank you!

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